Denoising algorithm of chaotic signal based on Lya-CEEMD-PR discrimination

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Abstract: Chaotic system has excellent performance and is widely used in secure communication, signal detection and other fields, but in practical application, chaotic signal is often polluted by noise. In order to effectively remove the noise in chaotic signal, the basic principle of CEEMD-PR (Complementary Ensemble Empirical Mode Decomposition Partial Reconstruction) is studied, and a CEEMD-PR denoising algorithm based on Lyapunov exponent discrimination (Lya-CEEMD-PR) is proposed, which can effectively suppress the noise in chaotic signal. Compared with the existing method based on the minimum CMSE (continuous mean square error) criterion, it has better detection performance.

1. Introduction

Chaotic signal has a wide range of applications in many fields, but it will inevitably be polluted by noise in the actual use process, which brings difficulties to the calculation of chaotic signal parameters, the synchronization of chaotic signal, the generation of chaotic password and many other applications [1-5]. The chaotic signal has the similar random characteristic, the existing signal denoising algorithm is not ideal for the chaotic signal denoising, so it is necessary to study the chaotic signal denoising algorithm. The empirical mode decomposition (EMD) method was proposed in the 1990s, and has been improved continuously. After decades of development, EMD method has a good effect in the field of signal denoising, but there are still some problems in chaotic signal denoising [6-8].

On the basis of EMD decomposition, this paper proposes a method of Lya-CEEMD-PR chaotic signal de-noising, which has better de-noising performance compared with the existing method based on the minimum CMSE criterion discrimination.

2. The basic theory of CEEMD

2.1 The basic theory of EMD

The basic principle of EMD is to decompose the signal layer by layer, and finally obtain several IMF components and a residual. Each IMF must meet two conditions:

(1) The number of zero crossing points of each IMF is equal to the number of extreme points, or at most one difference between the two number;

(2) For any sampling point, the average value of the upper and lower envelope generated by local maximum and local minimum is equal to zero.

EMD method can decompose signals by four steps as follows:

(1) Firstly, all local maxima and local minima of the signal $x(t)$ are determined, and the upper and lower envelopes are generated by fitting all local maxima and local minima with cubic spline;

(2) Average the upper envelope and the lower envelope and record them as m_1, subtracting m_1 from the signal $x(t)$:

$$h_1(t) = x(t) - m_1 \tag{1}$$

(3) If the conditions of the IMF are met, it is the first stage IMF. Otherwise, repeat steps (1) and steps (2) as the original signal to be decomposed. Check whether the conditions of the IMF are met. If the conditions are not met, continue to decompose until the qualified IMF is obtained. The first stage IMF will be recorded as C_1.

(4) The first order IMF is subtracted from the original signal to be decomposed, the resulting residual amount is:

$$r_1 = x(t) - C_1 \tag{2}$$

Regard r_1 as the original signal, repeat steps (1) - (3) until the second order IMF component of the signal is decomposed, repeat the screening iteration operation times, and decompose the signal into n IMFs and a residual r_n. When r_n becomes a monotone function, the decomposition ends. Therefore, the expression after decomposition is:

$$x(t)=\sum_{i=1}^{n} C_i(t) + r_n(t) \tag{3}$$

2.2 The basic theory of CEEMD

The decomposition process of CEEMD can be divided into the following steps:

(1) A group of different Gaussian white noise $n_i(t)$ is added to the signal $x(t)$ to construct a new signal to be decomposed:

$$x_i^+(t) = x(t) + n_i(t) \tag{4}$$

$x_i^+(t)$ is a new signal to be decomposed, $i = 1, 2, \cdots, I$.

Take the opposite number of the amplitude of the group of Gaussian white noise and add them to the signal $x(t)$ to build a new signal to be decomposed:

$$x_i^-(t) = x(t) - n_i(t) \tag{5}$$

(2) The EMD method is used to decompose the new signal $x_i^+(t)$. The components $imf_{ij}^+(t)$ and residual components $r_{in}^+(t)$ are obtained as follows:

$$x_i^+(t) = \sum_{j=1}^{n} imf_{ij}^+(t) + r_{in}^+(t) \tag{6}$$

Where, $j = 1, 2, \cdots, n$. n is the number of IMFs obtained from each decomposition. Similarly, the EMD method is used to decompose the new signal $x_i^-(t)$, and the results are as follows:

$$x_i^-(t) = \sum_{j=1}^{n} imf_{ij}^-(t) + r_{in}^-(t) \tag{7}$$

(3) The same order IMFs obtained from each decomposition is arithmetic averaged to obtain the IMF of signal $x^+(t)$. The residual components of each decomposition are arithmetically averaged to obtain the residual component of signal $x^+(t)$:

$$imf_j^+(t) = \frac{1}{I}\sum_{i=1}^{I} imf_{ij}^+(t) \tag{8}$$

$$r_n^+(t) = \frac{1}{I}\sum_{i=1}^{I} r_{in}^+(t) \tag{9}$$

In the same way, we can get:

$$imf_j^-(t) = \frac{1}{I}\sum_{i=1}^{I} imf_{ij}^-(t) \tag{10}$$

$$r_n^-(t) = \frac{1}{I}\sum_{i=1}^{I} r_{in}^-(t) \tag{11}$$

(4) The expression of signal reconstruction of CEEMD is as follows:

$$x(t) = \frac{1}{2}\sum_{j=1}^{n}[imf_j^+(t) + imf_j^-(t)] + \frac{1}{2}[r_n^+(t) + r_n^-(t)] \tag{12}$$

3. Denoising algorithm of chaotic signal based on Lya-CEEMD-PR discrimination

CEEMD decomposition is carried out for the noisy signal. The signal component is mainly concentrated in the low frequency component after decomposition, and the noise is mainly concentrated in the high frequency part. The main process of the signal denoising algorithm based on CEEMD-PR is as follows:

(1) CEEMD method is used to decompose the noisy signal, and the corresponding IMF component and residual component are obtained;

(2) The IMFs component was divided into two groups, one group had higher frequency, mainly noise part, the other group had lower frequency, mainly signal part;

(3) The noise with higher frequency is discarded, and the low frequency group and residual component which mainly contain signal components are selected to reconstruct the signal, and the denoised signal is obtained.

We take Lorenz chaotic signal as an example. Lorenz chaotic system is as follows:

$$\begin{cases} \dot{x}_1 = -\sigma(x_1 - x_2) \\ \dot{x}_2 = \gamma x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 = x_1 x_2 - b x_3 \end{cases} \tag{13}$$

Among them, σ、γ、b is the system parameter, when $\sigma = 10$, $\gamma = 28$, $b = 8/3$, the system enters the chaotic state. Take the signal x_1 in Lorenz chaotic system as an example, add Gauss white noise with signal-to-noise ratio of - 5dB, 0dB, 5dB and 10dB respectively, and decompose it by CEEMD. The Lyapunov exponent corresponding to IMF components of different orders is obtained as follows:

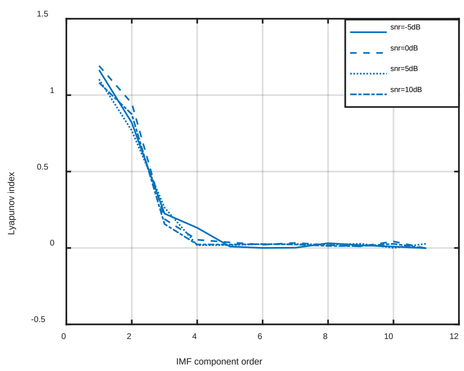

Figure 1. Lyapunov exponents after EMD decomposition of chaotic signals

It can be seen from Figure 1 that with the increase of signal decomposition order, the Lyapunov index gradually decreases and tends to be stable. The Lyapunov index of the low-order IMFs are larger, so the IMFs with smaller Lyapunov index are selected for signal reorganization to recover the original signal. The decision criteria are as follows: the IMFs with Lyapunov index less than 0.1 are selected for reconstruction to get the denoising signal.

Comparing the existing method with the method proposed in this paper, taking the denoising of noisy Lorenz chaotic system as an example, the simulation results are as follows:

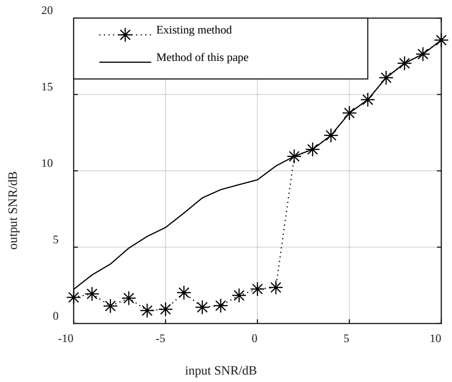

Figure 2. Comparison of denoising performance of two methods

It can be seen from Figure2 that: when the SNR is greater than 2dB, the existing method has the same performance as the method proposed in this paper; when the SNR is less than 2dB, the existing method's denoising performance drops rapidly, while the method proposed in this paper still has better denoising performance, and the highest output SNR is 7.5dB higher than the existing method.

4. Conclusion

In this paper, the chaos denoising algorithm based on Lya-CEEMD-PR discrimination is studied. Simulation results show that it has better denoising performance than the existing method.

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